



## 2018-2019 Curriculum Guide

November 1- December 7

### Eureka

Module 3: *Place Value, Counting and Comparison of Numbers to 1000*



ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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## Module 3 Performance Overview

- Students experience that 1 hundred is equal to both 100 ones and 10 tens. Likewise, 1 thousand is equal to both 100 tens and 10 hundreds. The efficiency of place value and base ten numbers comes to life as students repeatedly bundle 10 ones to make 1 ten and subsequently bundle 10 tens to make 1 hundred.
- In Topic B, students practice counting by ones and skip-counting by tens and hundreds. They start off with simple counting by ones and tens from 90 to 124 and 124 to 220. They then count by ones, tens, and hundreds from 200 to 432 and from 432 to 1,000. They apply their new counting strategies to solve a change unknown word problem. Students think in terms of getting to a ten or getting to a hundred. They also identify whether ones, tens, or hundreds are the appropriate unit to count efficiently and effectively.
- In Topic C, the teaching sequence opens with students counting on the place value chart by ones from 0 to 124, bundling larger units as possible. They represent three-digit numbers as number bonds and gain fluency in expressing numbers in unit form (3 hundreds 4 tens 3 ones), in word form, and on the place value chart. Students then count up by hundreds, tens, and ones, leading them to represent numbers in expanded form. The commutative property or “switch around rule” allows them to change the order of the units. They practice moving fluidly between word form, unit form, and expanded form.
- In Topic E, students transition to the more abstract number disks that will be used through Grade 5 when modeling very large and very small numbers. The three representations: bundles, money, and disks, each play an important role in the students’ deep internalization of the meaning of each unit on the place value chart. Finally, students evaluate numbers in unit form with more than 9 ones or tens, for example, 3 hundreds 4 tens 15 ones and 2 hundreds 15 tens 5 ones.
- Number disks make comparison of numbers very easy. More than and less than lead to the addition and subtraction in the next module. Students compare using the symbols  $<$ ,  $>$ , and  $=$  on the place value chart. Next, students advance to comparing different forms, and finally, they apply their comparison and place value skills to order more than two numbers in different forms.
- The module closes with questions such as, “What number is 10 less than 402?” and “What number is 100 more than 98?”. The language component of this segment is essential, too. Students need to be encouraged to use their words to make statements such as, “452 is 10 less than 462 and 100 less than 562.” This allows for greater understanding of comparison word problems.

**Module 3: Place Value, Counting and Comparison of Numbers to 1000**

**Pacing:**

November 1- December 7

**25 Days**

<b>Topic</b>	<b>Lesson</b>	<b>Student Lesson Objective/ Supportive Videos</b>
<b>Topic A:</b> Forming Base Ten Units of Ten, a Hundred and a Thousand	Lesson 1	Bundle and count ones, tens, and hundreds to 1,000.  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 2	Count up and down between 100 and 220 using ones and tens.  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic B:</b> Understanding Place Value of One, Ten, and a Hundred	Lesson 3	Count up and down between 90 and 1,000 using ones, tens, and hundreds  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 4	Count up to 1,000 on the place value chart  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic C:</b> Three-Digit Numbers in Unit, Standard, Expanded and Word Forms	Lesson 5	Write base ten three-digit numbers in unit form; show the value of each digit  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 6	Write base ten numbers in expanded form  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 7	Write, read, and relate base ten numbers in all forms  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic D:</b> Modeling Base Ten Numbers Within 1000 with Money	Lesson 8	Count the total value of \$1, \$10, and \$100 bills up to \$1,000  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 9	Count from \$10 to \$1,000 on the place value chart and the empty number line.  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	<b>Mid-Module Assessment</b> <b>Nov 16-19, 2018</b>	
<b>Topic E:</b> Modeling	Lesson 11	Count the total value of ones, tens, and hundreds with place value disks.  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 12	Change 10 ones for 1 ten, 10 tens for 1 hundred, and 10 hundreds for 1 thousand

Numbers Within 1000 with Place Value Disks		<a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 13	Read and write numbers within 1,000 after modeling with place value disks  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 14	Model numbers with more than 9 ones or 9 tens; write in expanded, unit, standard, and word forms  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 15	Explore a situation with more than 9 groups of ten  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic F:</b> Comparing Two Three-Digit Numbers	Lesson 16	Compare two three-digit numbers using $<$ , $>$ and $=$ .  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 17 & 18	Compare two three-digit numbers using $<$ , $>$ , and $=$ when there are more than 9 ones or 9 tens. Order numbers in different forms.  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a> <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic G:</b> Finding 1, 10, and 100 More	Lesson 19	Model and use language to tell about 1 more and 1 less, 10 more and 10 less, and 100 more and 100 less.  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 20	Lesson 20: Model 1 more and 1 less, 10 more and 10 less, and 100 more and 100 less when changing the hundreds place.  <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>End-Module Assessment</b> <b>December 6-7, 2018</b>		

## NJSLS Standards:

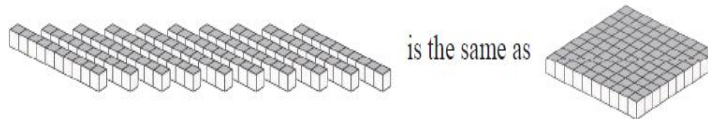
### ***Module 3: Place Value, Counting and Comparison of Numbers to 1000***

#### **2.NBT.1**

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: (See 2.NBT.1a & b)

- a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
- b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Second Grade students extend their base-ten understanding to hundreds as they view 10 tens as a unit called a “hundred”. They use manipulative materials and pictorial representations to help make a connection between the written three-digit numbers and hundreds, tens, and ones.



As in First Grade, Second Graders’ understanding about hundreds also moves through several stages: **Counting By Ones**; **Counting by Groups & Singles**; and **Counting by Hundreds, Tens and Ones**.

**Counting By Ones:** At first, even though Second Graders will have grouped objects into hundreds, tens and left-overs, they rely on counting all of the individual cubes by ones to determine the final amount. It is seen as the only way to determine how many.

**Counting By Groups and Singles:** While students are able to group objects into collections of hundreds, tens and ones and now tell how many groups of hundreds, tens and left-overs there are, they still rely on counting by ones to determine the final amount. They are unable to use the groups and left-overs to determine how many.

Example:

**Teacher:** How many blocks do you have?

**Student:** I have 3 hundreds, 4 tens and 2 left-overs.

**Teacher:** Does that help you know how many? How many do you have?

**Student:** Let me see. 100, 200, 300... ten, twenty, thirty, forty. So that’s 340 so far. Then 2 more. 342.

**Counting by Hundreds, Tens & Ones:** Students are able to group objects into hundreds, tens and ones, tell how many groups and left-overs there are, and now use that information to tell how many. Occasionally, as this stage becomes fully developed, second graders rely on counting to “really” know the amount, even though they may have just counted the total by groups and left-overs.

Example:

**Teacher:** How many blocks do you have?

**Student:** I have 3 hundreds, 4 tens and 2 left-overs.

**Teacher:** Does that help you know how many? How many do you have?

**Student:** Yes. That means that I have 342.

**Teacher:** Are you sure?

**Student:** Um. Let me count just to make sure. 100, 200, 300,...340, 341, 342. Yes. I was right. There are 342 blocks.

Understanding the value of the digits is more than telling the number of tens or hundreds. Second Grade students who truly understand the position and place value of the digits are also able to confidently model the number with some type of visual representation. Others who seem like they know, because they can state which number is in the tens place, may not truly know what each digit represents.

Example: Student Mastered

**Teacher:** What is this number? 726

**Student:** Seven hundred sixteen.

**Teacher:** Make this amount using your place value cards.

**Student:** *Uses 7 hundreds card, 2 ten cards and 6 singles.*

**Teacher:** *Pointing to the 6,* Can you show me where you have this?

**Student:** *Points to the 6 singles.*

**Teacher:** *Pointing to the 2,* Can you show me where you have this?

**Student:** *Points to the two tens.*

**Teacher:** *Pointing to the 7,* Can you show me where you have this?

**Student:** *Points to the 7 hundreds.*

Example: Student Not Yet Mastered

**Teacher:** What is this number? 726

**Student:** Seven hundred sixteen.

**Teacher:** Make this amount using your place value cards.

**Student:** *Uses 7 hundreds card, 2 ten cards and 6 singles.*

**Teacher:** *Pointing to the 6,* Can you show me where you have this?

**Student:** *Points to the 6 singles.*

**Teacher:** *Pointing to the 2,* Can you show me where you have this?

**Student:** *Points to two of the 6 singles (rather than two tens).*

Second Graders extend their work from first grade by applying the understanding that “100” is the same amount as 10 groups of ten as well as 100 ones. This lays the groundwork for the structure of the base-ten system in future grades.

Example:

**Teacher:** I have a pile of base-ten rods. Count out 12 please.

**Student:** Student gathers 12 ten-rods.

**Teacher:** How many cubes do you think you have?

**Student:** Makes an estimate.

**Teacher:** Count them to see.

**Student:** 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120. There's 120 here.

**Teacher:** So, do you think you have enough to make a 100?

**Student:** Yes.

**Teacher:** Go ahead and trade some in to make a 100.

**Student:** Student trades 10 rods for a 100 flat and leaves 2 tens remaining.

**Teacher:** What do you have now?

**Student:** I have 1 hundred and 2 tens.

**Teacher:** Does that help you know how many you have in all?

**Student:** Yes. 1 hundred and 2 tens is 120. There are 120 cubes here in all.

**2.NBT.2**

Count within 1000; skip-count by 5s, 10s, and 100s

Second Grade students count within 1,000. Thus, students “count on” from any number and say the next few numbers that come afterwards.

Example:

**What are the next 3 numbers after 498?** 499, 500, 501.

**When you count back from 201, what are the first 3 numbers that you say?** 200, 199, 198.

Second grade students also begin to work towards multiplication concepts as they skip count by 5s, by 10s, and by 100s. Although skip counting is not yet true multiplication because students don’t keep track of the number of groups they have counted, they can explain that when they count by 2s, 5s, and 10s they are counting groups of items with that amount in each group.

As teachers build on students’ work with skip counting by 10s in Kindergarten, they explore and discuss with students the patterns of numbers when they skip count. For example, while using a 100s board or number line, students learn that the ones digit alternates between 5 and 0 when skip counting by 5s. When students skip count by 100s, they learn that the hundreds digit is the only digit that changes and that it increases by one number..

**2.NBT.3**

Read and write numbers to 1000 using base-ten numerals, number names, and expanded form

Second graders read, write and represent a number of objects with a written numeral (number form or standard form). These representations can include snap cubes, place value (base 10) blocks, pictorial representations or other concrete materials. Please be cognizant that when reading and writing whole numbers, the word “and” should not be used (e.g., 235 is stated and written as “two hundred thirty-five).

Expanded form (125 can be written as  $100 + 20 + 5$ ) is a valuable skill when students use place value strategies to add and subtract large numbers in 2.NBT.7

**2.NBT.4**

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons

Second Grade students build on the work of **2.NBT.1** and **2.NBT.3** by examining the amount of hundreds, tens and ones in each number. When comparing numbers, students draw on the understanding that 1 hundred (the smallest three-digit number) is actually greater than any amount of tens and ones represented by a two-digit number. When students truly understand this concept, it makes sense that one would compare three-digit numbers by looking at the hundreds place first.

Students should have ample experiences communicating their comparisons in words before using symbols. Students were introduced to the symbols greater than ( $>$ ), less than ( $<$ ) and equal to ( $=$ ) in First Grade and continue to use them in Second Grade with numbers within 1,000.

Example: Compare these two numbers. 452 \_\_\_ 455

**Student A**

*Place Value*

452 has 4 hundreds 5 tens and 2 ones. 455 has 4 hundreds 5 tens and 5 ones. They have the same number of hundreds and the same number of tens, but 455 has 5 ones and 452 only has 2 ones. 452 is less than 455.

$452 < 455$

**Student B**

*Counting*

452 is less than 455. I know this because when I count up I say 452 before I say 455.

$452 < 455$

452 is less than 455.

While students may have the skills to order more than 2 numbers, this Standard focuses on comparing two numbers and using reasoning about place value to support the use of the various symbols

**M** : Major Content

**S**: Supporting Content

**A** : Additional Content



Common addition and subtraction.<sup>1</sup>

	<b>RESULT UNKNOWN</b>	<b>CHANGE UNKNOWN</b>	<b>START UNKNOWN</b>
<b>ADD TO</b>	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
<b>TAKE FROM</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	<b>TOTAL UNKNOWN</b>	<b>ADDEND UNKNOWN</b>	<b>BOTH ADDENDS UNKNOWN<sup>2</sup></b>
<b>PUT TOGETHER / TAKE APART<sup>3</sup></b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ , $5 - 3 = ?$	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? $5 = 0 + 5$ , $5 + 0 = 5$ , $1 + 4 = 5$ , $4 + 1 = 5$ , $2 + 3 = 5$ , $3 + 2 = 5$
<b>COMPARE</b>	<b>DIFFERENCE UNKNOWN</b>	<b>BIGGER UNKNOWN</b>	<b>SMALLER UNKNOWN</b>
	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ , $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ , $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$

<sup>1</sup> Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

<sup>2</sup> These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

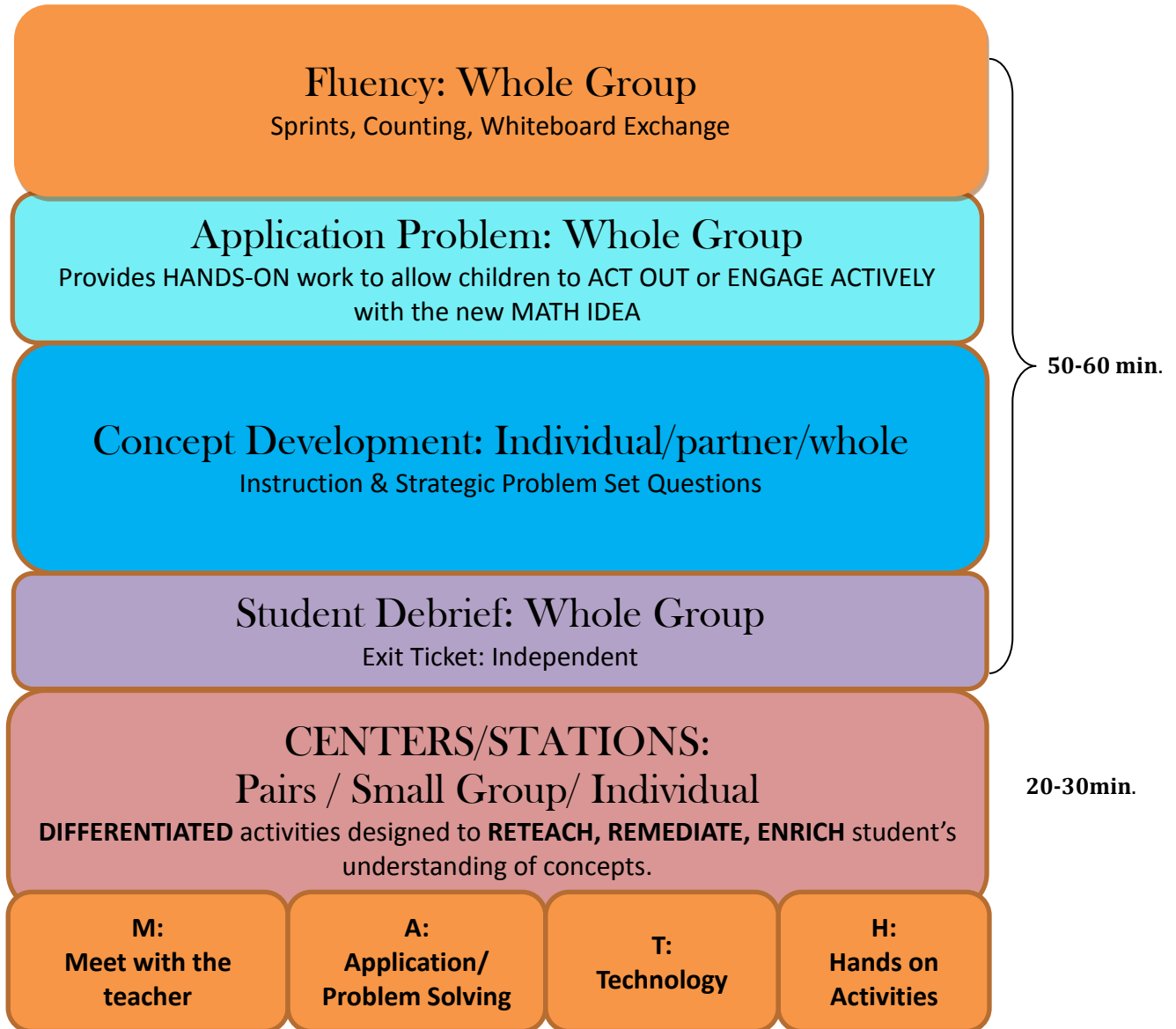
<sup>3</sup> Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

<sup>4</sup> For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

<http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/>

<b>Module 3 Assessment / Authentic Assessment Recommended Framework</b>			
<b>Assessment</b>	<b>CCSS</b>	<b>Estimated Time</b>	<b>Format</b>
<b><i>Eureka Math</i></b> <b><i>Module 3 : Place Value, Count to 1000</i></b>			
Authentic Assessment #1	2.NBT.3	30 mins	Individual
Optional Mid Module Assessment	2.NBT.1-4	1 Block	Individual
Optional End of Module Assessment (Standards Mastery)	2.NBT.1-4	1 Block	Individual
Grade 2 Interim Assessment 1 <b>10/29-11/12</b>	2.NBT.5 2.OA.1-2 2.MD.1-6	1 Block	Individual

# Second Grade Ideal Math Block



## Eureka Lesson Structure:

### Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

### Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

### Concept Development: (largest chunk of time)

#### Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

#### Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

### Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

**PARCC Evidence/Clarification Statements**

<b>CCSS</b>	<b>Evidence Statement</b>	<b>Clarification</b>	<b>Math Practices</b>
2.NBT.1a	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: 100 can be thought of as a bundle of ten tens — called a “hundred.”	i Tasks have “thin context” or no context.	MP 7,8
2.NBT.1b	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	i Tasks have “thin context” or no context	MP 7,8

## Number Talks Cheat Sheet

### **What does Number Talks look like?**

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

### **Communication**

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

### **Mental Math**

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

### **Thumbs Up**

- This is just a signal to let you know that you have given your students enough time to think about the problem
- It will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

### **Teacher as Recorder**

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

### **Purposeful Problems**

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

### **Starting Number Talks in your Classroom**

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

### **The teacher asks questions:**

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?
- What was the first thing your eyes saw, or your brain did?
- What are Number Talks and Why are they

Student Name: \_\_\_\_\_

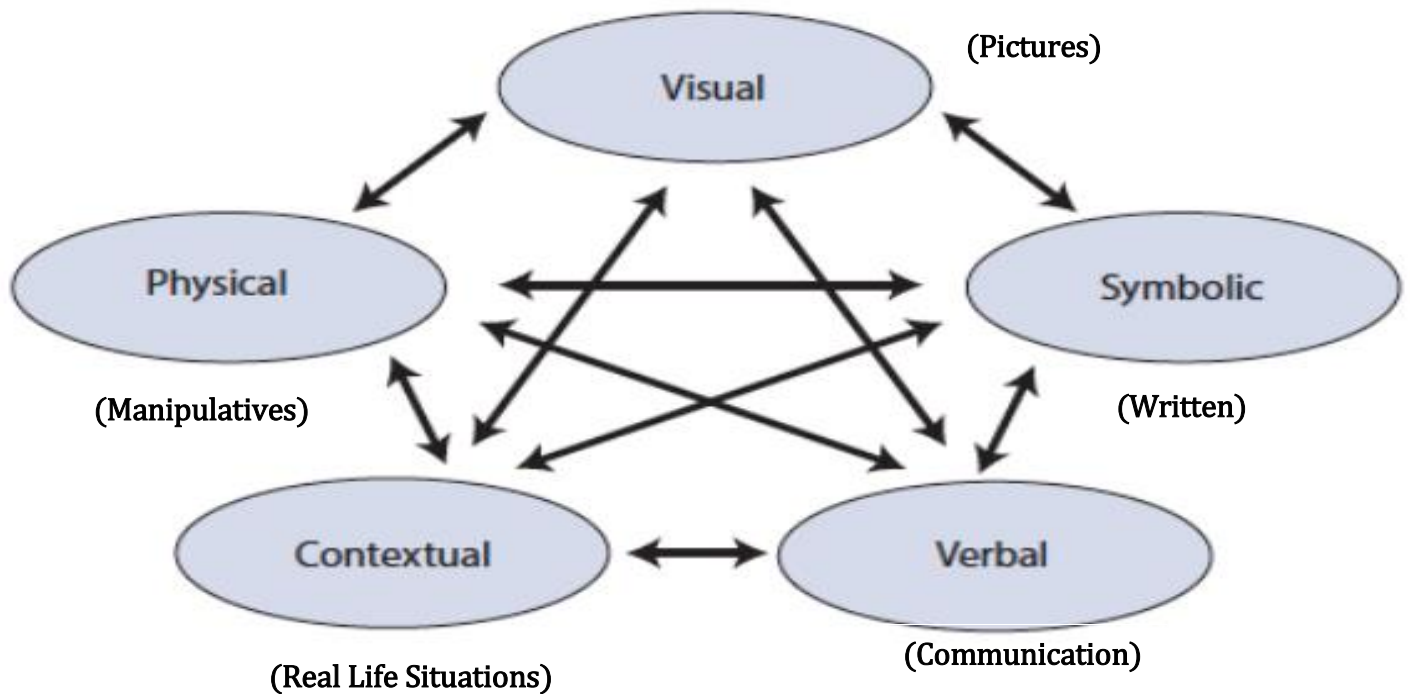
Task: \_\_\_\_\_

School: \_\_\_\_\_

Teacher: \_\_\_\_\_ Date: \_\_\_\_\_

"I CAN....."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a classmate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did.  I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my thinking.	

## Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

**Visual:** When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

**Physical:** The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

**Verbal:** Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.



**Symbolic:** Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

**Contextual:** A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

### **The Lesh Translation Model: Importance of Connections**

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

## Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

**Concrete:** “Doing Stage”: Physical manipulation of objects to solve math problems.

**Pictorial:** “Seeing Stage”: Use of imaged to represent objects when solving math problems.

**Abstract:** “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

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## Read, Draw, Write Process

**READ** the problem. Read it over and over.... And then read it again.

**DRAW** a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

**WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

## Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

### Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



The most  
important thing  
is to NEVER  
stop  
questioning

*Albert Einstein*

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

**100** questions that promote  
**Mathematical Discourse**

Help students **work together** to make sense of mathematics

- 1 What **strategy** did you use?
- 2 Do you **agree**?
- 3 Do you **disagree**?
- 4 Would you **ask the rest of the class** that question?
- 5 Could you **share your method** with the class?
- 6 What part of what he said **do you understand**?
- 7 Would someone like to **share** \_\_\_?
- 8 Can you **convince the rest of us** that your answer makes sense?
- 9 **What do others think** about what [student] said?
- 10 Can someone **retell or restate** [student]'s explanation?
- 11 Did you **work together**? In what way?
- 12 Would anyone like to **add to what was said**?
- 13 Have you **discussed** this with your group? With others?
- 14 Did anyone get a **different answer**?
- 15 **Where** would you go for **help**?
- 16 **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
- 17 How could you help another student **without telling them the answer**?
- 18 **How would you explain** \_\_\_ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

- 19 Is this a **reasonable answer**?
- 20 Does that make **sense**?
- 21 **Why** do you think that? Why is that true?
- 22 Can you **draw a picture or make a model** to show that?
- 23 **How** did you reach that conclusion?
- 24 Does anyone want to **revise** his or her answer?
- 25 **How were you sure** your answer was right?

Ready

## Help students learn to reason mathematically

- 26 How did you **begin** to think about this problem?
- 27 What is **another way** you could solve this problem?
- 28 How could you **prove** \_\_\_\_\_?
- 29 Can you **explain how your answer is different from or the same as** [student]'s answer?
- 30 Let's **break the problem into parts**. What would the parts be?
- 31 Can you **explain this part more specifically**?
- 32 Does that **always work**?
- 33 Can you think of a case where that **wouldn't work**?
- 34 How did you **organize** your information? Your thinking?

## Help students with problem comprehension

- 39 What is this problem about? What can you **tell me about it**?
- 40 Do you need to **define or set limits** for the problem?
- 41 How would you **interpret** that?
- 42 Could you **reword that in simpler terms**?
- 43 Is there something that can be **eliminated** or that is **missing**?
- 44 Could you **explain** what the problem is asking?
- 45 What **assumptions** do you have to make?
- 46 What do you **know** about this part?
- 47 Which words were **most important**? Why?

## Help students evaluate their own processes and engage in productive peer interaction

- 35 What do you need to do **next**?
- 36 What have you **accomplished**?
- 37 What are your **strengths and weaknesses**?
- 38 Was your **group participation appropriate and helpful**?



## Help students learn to **conjecture, invent, and solve problems**

- 48 What would happen if \_\_\_?
- 49 Do you see a **pattern**?
- 50 What are some **possibilities** here?
- 51 Where could you find the **information** you need?
- 52 How would you **check your steps** or your answer?
- 53 What **did not work**?
- 54 How is your solution method the **same as or different from** [student]'s method?
- 55 Other than retracing your steps, **how can you determine** if your answers are appropriate?
- 56 How did you **organize** the information? Do you have a **record**?
- 57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?
- 58 What have you tried? What **steps** did you take?
- 59 How would it look if you used this **model** or these **materials**?
- 60 How would you draw a **diagram or make a sketch** to solve the problem?
- 61 Is there **another possible answer**? If so, explain.
- 62 Is there **another way to solve** the problem?
- 63 Is there **another model** you could use to solve the problem?
- 64 Is there anything you've **overlooked**?
- 65 **How did you think** about the problem?
- 66 What was your **estimate or prediction**?
- 67 How **confident** are you in your answer?
- 68 **What else** would you like to know?
- 69 What do you think comes **next**?
- 70 Is the solution **reasonable**, considering the context?
- 71 Did you have a **system**? Explain it.
- 72 Did you have a **strategy**? Explain it.
- 73 Did you have a **design**? Explain it.





## Help students learn to connect mathematics, its ideas, and its application

- 74 What is the **relationship** between \_\_\_ and \_\_\_?
- 75 Have we ever solved a problem **like this before**?
- 76 What uses of mathematics did you find in the **newspaper** last night?
- 77 What is the **same**?
- 78 What is **different**?
- 79 Did you use skills or build on concepts that were **not necessarily mathematical**?
- 80 Which **skills or concepts** did you use?
- 81 What **ideas** have we explored before that were useful in solving this problem?

- 82 Is there a **pattern**?
- 83 **Where else** would this strategy be useful?
- 84 How does this **relate** to \_\_\_?
- 85 Is there a **general rule**?
- 86 Is there a **real-life situation** where this could be used?
- 87 How would your method work with **other problems**?
- 88 What other problem does this seem to **lead to**?

### Help students persevere

- 95 What was **one thing you learned** (or two, or more)?
- 96 Did you **notice any patterns**? If so, describe them.
- 97 What **mathematics topics** were used in this investigation?
- 98 What were the **mathematical ideas** in this problem?
- 99 What is mathematically **different about these two situations**?
- 100 What are the **variables** in this problem? What stays **constant**?

- 89 Have you tried making a **guess**?
- 90 **What else** have you tried?
- 91 Would **another method** work as well or better?
- 92 Is there **another way** to draw, explain, or say that?
- 93 Give me another **related problem**. Is there an easier problem?
- 94 How would you **explain** what you know right now?

### Help students focus on the mathematics from activities

## **Conceptual Understanding**

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

## **Procedural Fluency**

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

## **Math Fact Fluency: Automaticity**

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

### **K-2 Math Fact Fluency Expectation**

**K.OA.5** Add and Subtract within 5.

**1.OA.6** Add and Subtract within 10.

**2.OA.2** Add and Subtract within 20.



## **Math Fact Fluency: Fluent Use of Mathematical Strategies**

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

**1.OA.6** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g.,  $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$ );
- decomposing a number leading to a ten (e.g.,  $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$ );
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

**2.NBT.7** Add and subtract within 1000, using concrete models or drawings and strategies based on:

- place value,
- properties of operations, and/or
- the relationship between addition and subtraction;

## Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

## Mathematical Proficiency

To be mathematically proficient, a student must have:

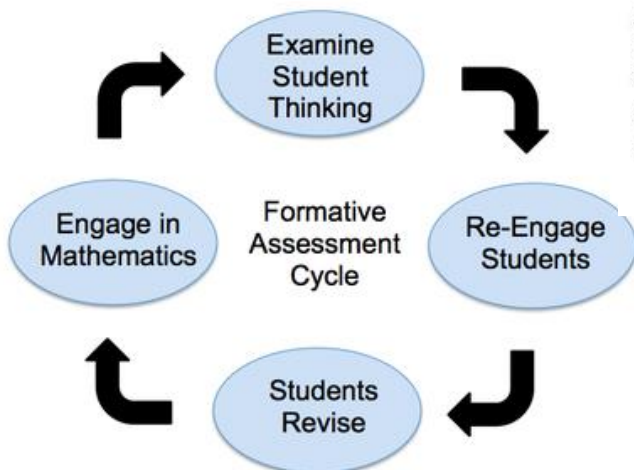
- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

## Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

*Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.*

(Wiliam 2007, pp. 1054; 1091)



## Connections to the Mathematical Practices

### **Student Friendly Connections to the Mathematical Practices**

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

**Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Students can make sense of the meaning of the task and find an entry point or a way to start the task. Second Grade students continue to use concrete manipulatives and pictorial representations as well as mental mathematics. Second Grade students continue to solve the task. Lastly, mathematically proficient students complete a task by asking themselves the question

relationships while solving tasks. This involves two processes- decontextualizing and contextualizing. In Second Grade, students are asked, "There are 27 children in the cafeteria. How many students are in the cafeteria?" Second Grade students translate that situation into an equation. Then, students can refer to the context of the task to determine that they need to subtract 19 since 19 children leave. The

previously established solutions to construct viable arguments about mathematics. During discussions about problem situations, and after working on the task, students can discuss and critique each others' reasoning and strategies, citing similarities

equations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem. Students are able to create an appropriate problem situation from an equation. For example, students are expected to create a problem situation for the equation "How many children are in the machine?"

appropriately. These tools may include snap cubes, place value (base ten) blocks, hundreds number boards, number lines, and other manipulatives, which support conceptual understanding and higher-order thinking skills. During classroom instruction, students can explain why a yardstick is more appropriate to use than a ruler.

on, calculations, and measurements. In all mathematical tasks, students in Second Grade communicate clearly, using appropriate language. When measuring an object, care is taken to line up the tool correctly in order to get an accurate measurement. During tasks involving

structures in the number system and other areas of mathematics. For example, students notice number patterns within the base ten system. Students work with the idea that 10 ones equals a ten, and 10 tens equals 1 hundred. In addition, Second Grade students solve problems such as, "How much more do I need to add to 33 to get to 50?"

problem structures when solving mathematical tasks. For example, after solving two digit addition problems by decomposing, students use strategies to be more efficient in computations, including doubles strategies and making a ten. Lastly, while solving a

## Effective Mathematics Teaching Practices

**Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

**Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

**Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

**Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

**Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

**Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

**Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

## **5 Practices for Orchestrating Productive Mathematics Discussions**

Practice	Description/ Questions
1. Anticipating	<p>What strategies are students likely to use to approach or solve a challenging high-level mathematical task?</p> <p>How do you respond to the work that students are likely to produce?</p> <p>Which strategies from student work will be most useful in addressing the mathematical goals?</p>
2. Monitoring	<p>Paying attention to what and how students are thinking during the lesson.</p> <p>Students working in pairs or groups</p> <p>Listening to and making note of what students are discussing and the strategies they are using</p> <p>Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)</p>
3. Selecting	<p>This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.</p>
4. Sequencing	<p>What order will the solutions be shared with the class?</p>
5. Connecting	<p>Asking the questions that will make the mathematics explicit and understandable.</p> <p>Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.</p>

## MATH CENTERS/ WORKSTATIONS

*Math workstations* allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

### Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated.** If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.

In her book, she leads you step-by-step through the process of implementing work stations.

**MATH WORKSTATION INFORMATION CARD**

**Math Workstation:** \_\_\_\_\_

**Time:**

**NJSLS:**

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**Objective(s):** By the end of this task, I will be able to:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

**Task(s):**

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

**Exit Ticket:**

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_



# MATH WORKSTATION SCHEDULE

Week of: \_\_\_\_\_

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	<b>BASED ON CURRENT OBSERVATIONAL DATA</b>
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	
	Group ____	Group ____	Group ____	Group ____	

## INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

## Second Grade PLD Rubric

<b>Got It</b>		<b>Not There Yet</b>		
Evidence shows that the student essentially has the target concept or big math idea.		Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
<b>PLD Level 5: 100% Distinguished command</b>	<b>PLD Level 4: 89% Strong Command</b>	<b>PLD Level 3: 79% Moderate Command</b>	<b>PLD Level 2: 69% Partial Command</b>	<b>PLD Level 1: 59% Little Command</b>
<p>Student work shows <b>distinct levels of understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> a <b>complete response</b> based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes an <b>efficient and logical progression</b> of mathematical reasoning and understanding.</p>	<p>Student work shows <b>strong levels of understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> a <b>complete response</b> based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes a <b>logical progression</b> of mathematical reasoning and understanding.</p>	<p>Student work shows <b>moderate levels of understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> a <b>complete response</b> based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes a <b>logical but incomplete progression</b> of mathematical reasoning and understanding. Contains <b>minor errors</b>.</p>	<p>Student work shows <b>partial understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> an <b>incomplete response</b> based on student's attempts of explanations/ reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes an <b>incomplete or illogical progression</b> of mathematical reasoning and understanding.</p>	<p>Student work shows <b>little understanding</b> of the mathematics.</p> <p>Student <b>attempts to construct</b> and <b>communicates</b> a response using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes <b>limited evidence of the progression</b> of mathematical reasoning and understanding.</p>
<b>5 points</b>	<b>4 points</b>	<b>3 points</b>	<b>2 points</b>	<b>1 point</b>

## DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

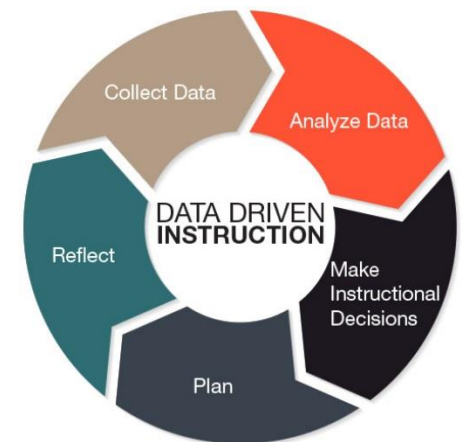
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



**Data Analysis Form**

School: \_\_\_\_\_

Teacher: \_\_\_\_\_

Date: \_\_\_\_\_

Assessment: \_\_\_\_\_

NJSLS: \_\_\_\_\_

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

## **MATH PORTFOLIO EXPECTATIONS**

The **Student Assessment Portfolios for Mathematics** are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSL and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

### **K-2 GENERAL PORTFOLIO EXPECTATIONS:**

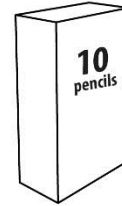
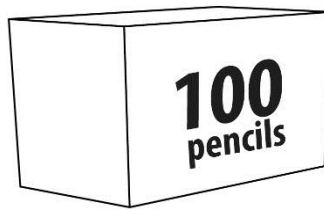
- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews<sup>1</sup>."
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended*.
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)<sup>2</sup>.
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

## **GRADES K-2**

### **Student Portfolio Review**

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfoli

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**Pencils come in cases of 100, packs of 10, or as single pencils.**

**Write the number of pencils that you have in number form and expanded form.**

	<b>Number Form</b>	<b>Expanded Form</b>
a) 6 singles, 9 packs, and 4 cases		
b) 1 pack, 3 singles, and 7 cases		
c) 8 cases, 2 singles, and 3 packs		
d) 0 packs, 5 cases, and 0 singles		
e) 1 case, 0 singles and 4 packs		
f) 5 packs, 7 cases, and 0 singles		
g) 0 packs, 1 cases, and 9 singles		

## Pencils

<b>Domain</b>	Number and Operations in Base Ten
<b>Cluster</b>	Understand place value.
<b>Standard(s)</b>	<b>2.NBT.3.</b> Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
<b>Materials</b>	SF, Pencil, Paper
<b>Task</b>	Provide materials to the student. Read the problem to the student. <i>Pencils come in cases of 100, packs of 10, or as single pencils. Write the number of pencils that you have in number form and expanded form.</i> <ol style="list-style-type: none"> <li>a) 6 singles, 9 packs, and 4 cases</li> <li>b) 1 pack, 3 singles, and 7 cases</li> <li>c) 8 cases, 2 singles, and 3 packs</li> <li>d) 0 packs, 5 cases, and 0 singles</li> <li>e) 1 case, 0 singles and 4 packs</li> <li>f) 5 packs, 7 cases, and 0 singles</li> <li>g) 1 case, 0 packs, and 9 singles</li> </ol>

### Continuum of Understanding

<b>Developing Understanding</b>	<ul style="list-style-type: none"> <li>Errors are made either in writing the number form or expanded form.</li> </ul>
<b>Complete Understanding</b>	<ul style="list-style-type: none"> <li>Correctly solves each item in both number form and expanded form.</li> </ul> <p><u>Solutions:</u></p> <ol style="list-style-type: none"> <li>a) 496, <math>400 + 90 + 6</math></li> <li>b) 713, <math>700 + 10 + 3</math></li> <li>c) 832, <math>800 + 30 + 2</math></li> <li>d) 500, <math>500 + 0 + 0</math></li> <li>e) 140, <math>100 + 40 + 0</math></li> <li>f) 750, <math>700 + 50 + 0</math></li> <li>g) 109, <math>100 + 0 + 9</math></li> </ol>

<b>Level 5: Distinguished Command</b>	<b>Level 4: Strong Command</b>	<b>Level 3: Moderate Command</b>	<b>Level 2: Partial Command</b>	<b>Level 1: No Command</b>
Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using : <ul style="list-style-type: none"> <li>Concepts of reading and writing numbers in the base ten number system</li> <li>Strategies based on place value</li> </ul>	Student correctly answers and clearly constructs and communicates a complete response with one minor calculation error based on explanations/reasoning using: <ul style="list-style-type: none"> <li>Concepts of reading and writing numbers in the base ten number system</li> <li>Strategies based on</li> </ul>	Student answers, clearly constructs, and communicates a complete response with minor calculation errors based on explanations/reasoning using: <ul style="list-style-type: none"> <li>Concepts of reading and writing numbers in the base ten number system</li> <li>Strategies based on place value</li> </ul>	Student answers, clearly constructs, and communicates a complete response with major calculation errors and/or conceptual errors based on explanations/reasoning using: <ul style="list-style-type: none"> <li>Concepts of reading and writing numbers in the base ten number system</li> <li>Strategies based on</li> </ul>	The student shows no work or justification.

<p>Response includes an <b>efficient</b> and logical progression of steps.</p>	<p>place value Response includes a <b>logical</b> progression of steps</p>	<p>Response includes a <b>logical but incomplete</b> progression of steps. Minor calculation errors.</p>	<p>place value Response includes an <b>incomplete or illogical</b> progression of steps.</p>	
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## Resources

### **Great Minds**

<https://greatminds.org/>

### **Embarc**

<https://embarc.online/>

**Number Book Assessment** Link: <http://investigations.terc.edu/>

**Model Curriculum-** <http://www.nj.gov/education/modelcurriculum/>

**Georgia Department of Education: Games to be played at centers with a partner or small group.** <http://ccgpsmathematicsk-5.wikispaces.com/Kindergarten>

**Engage NY: \*For additional resources to be used during centers or homework.**

<https://www.engageny.org/sites/default/files/resource/attachments/math-gk-m1-full-module.pdf>

**Add/ Subtract Situation Types:** Darker Shading indicates Kindergarten expectations

<https://achievethecore.org/content/upload/Add%20Subtract%20Situation%20Types.pdf>

**Math in Focus PD Videos:** <https://www->

[k6.thinkcentral.com/content/hsp/math/hspmath/common/mif\\_pd\\_vid/9780547760346\\_te/index.html](https://www-thinkcentral.com/content/hsp/math/hspmath/common/mif_pd_vid/9780547760346_te/index.html)

**Number Talk/Strings:** <https://elementarynumbertalks.wordpress.com/second-grade-number-talks/>

## Suggested Literature

*Fish Eyes* by, Lois Ehlert

*Ten Little Puppies* by, Elena Vazquez

*Zin! Zin! Zin! A Violin!* by, Lloyd Moss

*My Granny Went to the Market* by, Stella Blackstone and Christopher Corr

*Anno's Counting Book* by, Mitsumasa Anno

*Chicka, Chicka, 1,2,3* by, Bill Martin Jr.; Michael Sampson; Lois Ehlert

*How Dinosaurs Count to 10* by Jane Yolen and Mark Teague

*10 Little Rubber Ducks* by Eric Carle

*Ten Black Dots* by Donald Crews

*Mouse Count* by Ellen Stoll Walsh

*Count!* by Denise Flemin

# 21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see **21<sup>st</sup> Century Career Ready Practices** .

## References

“Eureka Math” *Great Minds*. 2018 < <https://greatminds.org/account/products>>